Minimizing the Regret of an Influence Provider

Yipeng Zhang, Yuchen Li, Zhifeng Bao, Baihua Zheng, H. V. Jagadish

Out-Of-Home Advertising

Out-of-home advertising (OOH) is any visual advertising media found outside of the home.

Why this problem interest us?

- Business perspective
  - USD 6.13 million in 2020 → USD 15.03 million by 2026[1]
- Academic perspective
  - Problem Definition
    - Existing studies: Single advertiser
    - Our work: Host (Influence provider)
  - Hardness

1. https://www.mordorintelligence.com/industry-reports/digital-ooh-market

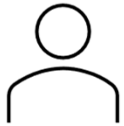
For Advertiser – Existing Work

I provide - Billboards
I earn - Money

Host

Billboard	Influence	Cost
Billboard 1	30	100
Billboard 2	50	200
Billboard 3	100	300
……		

I pay - Money
I need - Influence



Advertiser

For Advertiser – Existing Work

Advertiser needs billboards to advertising (under budget).

Given budget B

Host owns billboards

Given a set of billboards S; each of them has a cost

Find billboards to advertising for this advertiser under her budget, which have the best effectiveness.

Find a subset billboard $S' \in S$, that achieves $\mathrm{argmax}\, I(S')$ while the total cost is not larger than B, where $I()$ is a given influence module

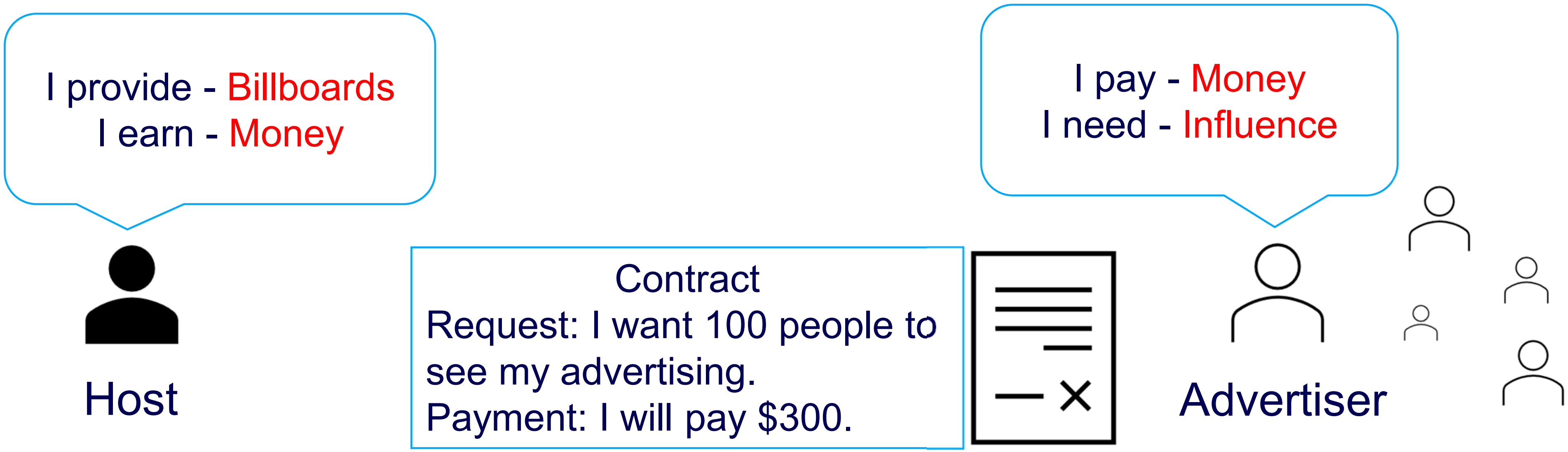
For Advertiser – Existing Work

Find billboards to advertising for one advertiser under her budget, which have the maximum influence.

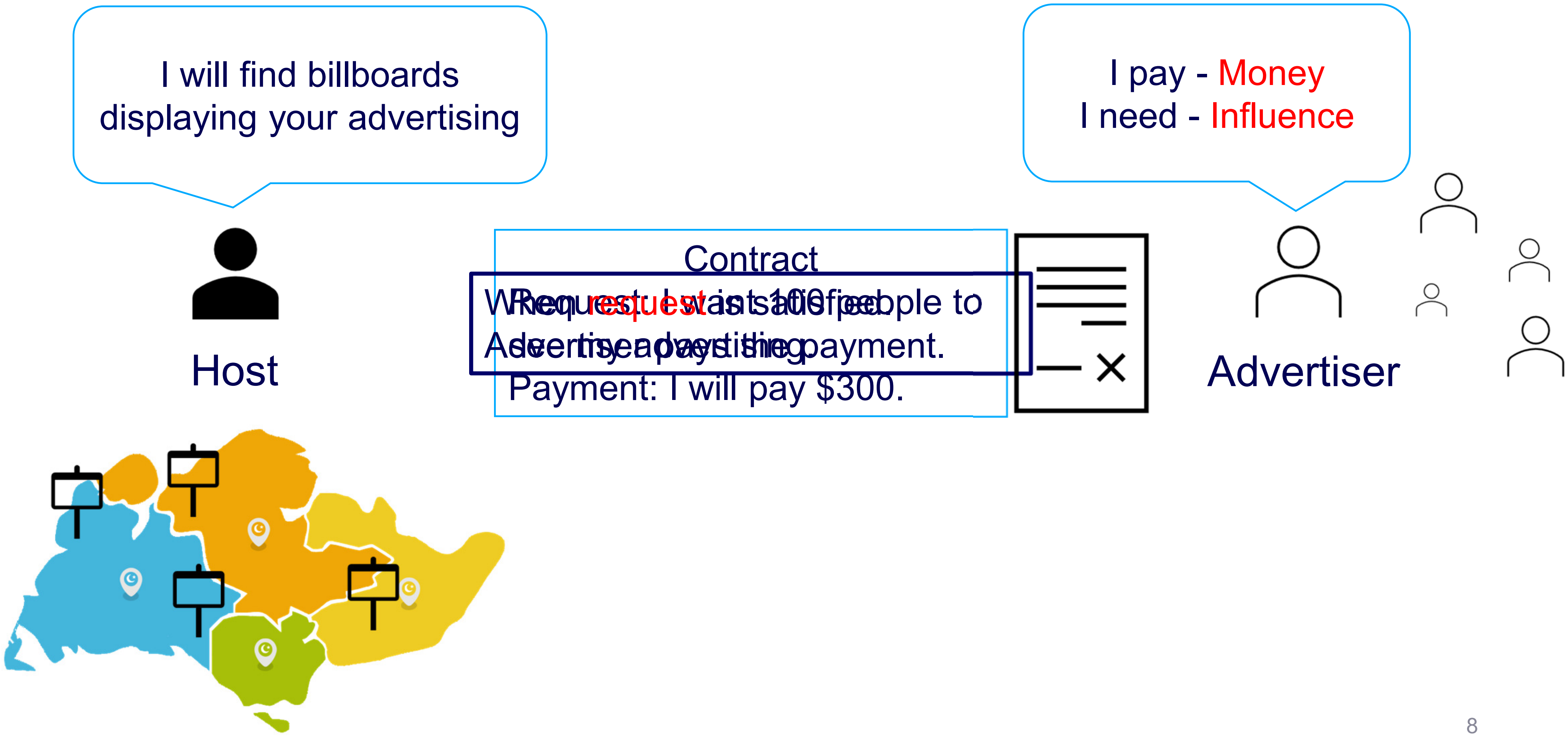
Real-world:

1. Host needs to deal with multiple advertisers.
2. Each advertiser has a demanded influence.

Host - Advertisers



Host - Advertisers



For Host – Our Work

Host owns billboards; Advertisers request influence service, each advertiser has a budget and a demanded influence

Input: billboard set S, advertisers set A, each $a_i \in A$ has a budget L_i and a demanded influence I_i

Find billboards for each advertiser, which can achieve the demanded influence, so that maximize the host's profit.

Output: billboard sets $\mathrm{S_i} \in S$, that achieves $\mathrm{argmax} \sum_{i=1}^{|A|} R(S_i)$, $R()$ is how to measure the profit.

For Host – Our Work

What is profit?

Profit = Total payment from all advertisers

For Host – Our Work

Provides Influence and earns the Profit

Contract
Request: Influence 300 audiences.
Payment: Pay $300.

Host

Advertiser

Scenario 1

Provides Influence and earns the Profit

You break the contract!
I will only pay 100$!

Host

Advertiser

200 audiences
are influenced

$100

Unsatisfied Penalty

Case 1

Scenario 2

Provides Influence and earns the Profit

Host $100

Advertiser

Unsatisfied Penalty

Case 1

I only get $300.
The extra audience is free service.

Host

400 audiences are influenced

Advertiser

$300

Excessive Influence

Case 2

Scenario 2

Provides Influence and earns the Regret

Minimize Profit

1. Unsatisfied Penalty
2. Excessive Influence

Unsatisfied Penalty

Case 1

Excessive Influence

Case 2

For Host – Our Work

$$\mathrm{argmin} \sum_{i=1}^{|A|} R(S_i)$$

~~Profit = Total payment from all Advertisers~~

Regret = Unsatisfied Penalty + Excessive Influence

Minimize Regret ⊆ Maximize Profit

MROAM

Minimizing Regret for the OOH Advertising Market

For Host – Our Work

$$\text{argmin} \sum_{i=1}^{|A|} R(S_i)$$

$$R(S_i) = \begin{cases} L_i\left(1 - \frac{\gamma \cdot I(S_i)}{I_i}\right), & if \;\; a_i.I_i > I(S_i) \\ L_i \frac{I(S_i) - I_i}{I_i}, & otherwise \end{cases}$$

Unsatisfied Penalty

Excessive Influence

L_i : budget of a_i
I_i : demanded influence of a_i
$I(S_i)$: billboards influence assigned to a_i

Minimizing Regret for the OOH Advertising Market problem (MROAM)

1. MROAM is non-monotone and non-submodular
   - Any greedy-based algorithm is not applicable
2. MROAM problem is NP-hard to approximate within any constant factor

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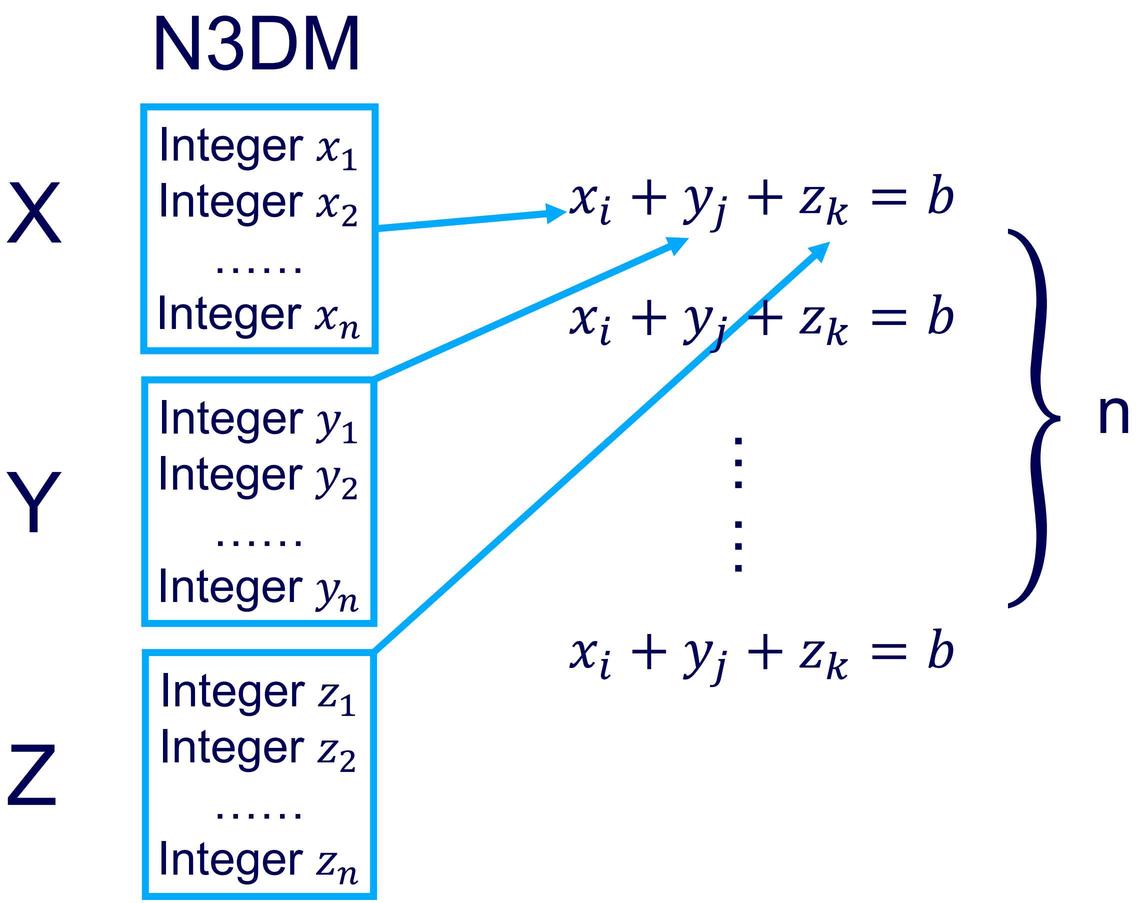
Baseline - Synchronous Greedy

Randomized local search framework

(1) advertiser-driven local search
(2) billboard-driven local search.

Hardness - Mapping



Hardness - Mapping

N3DM

X
- Integer x_1
- Integer x_2
- ……
- Integer x_n

Y
- Integer y_1
- Integer y_2
- ……
- Integer y_n

Z
- Integer z_1
- Integer z_2
- ……
- Integer z_n

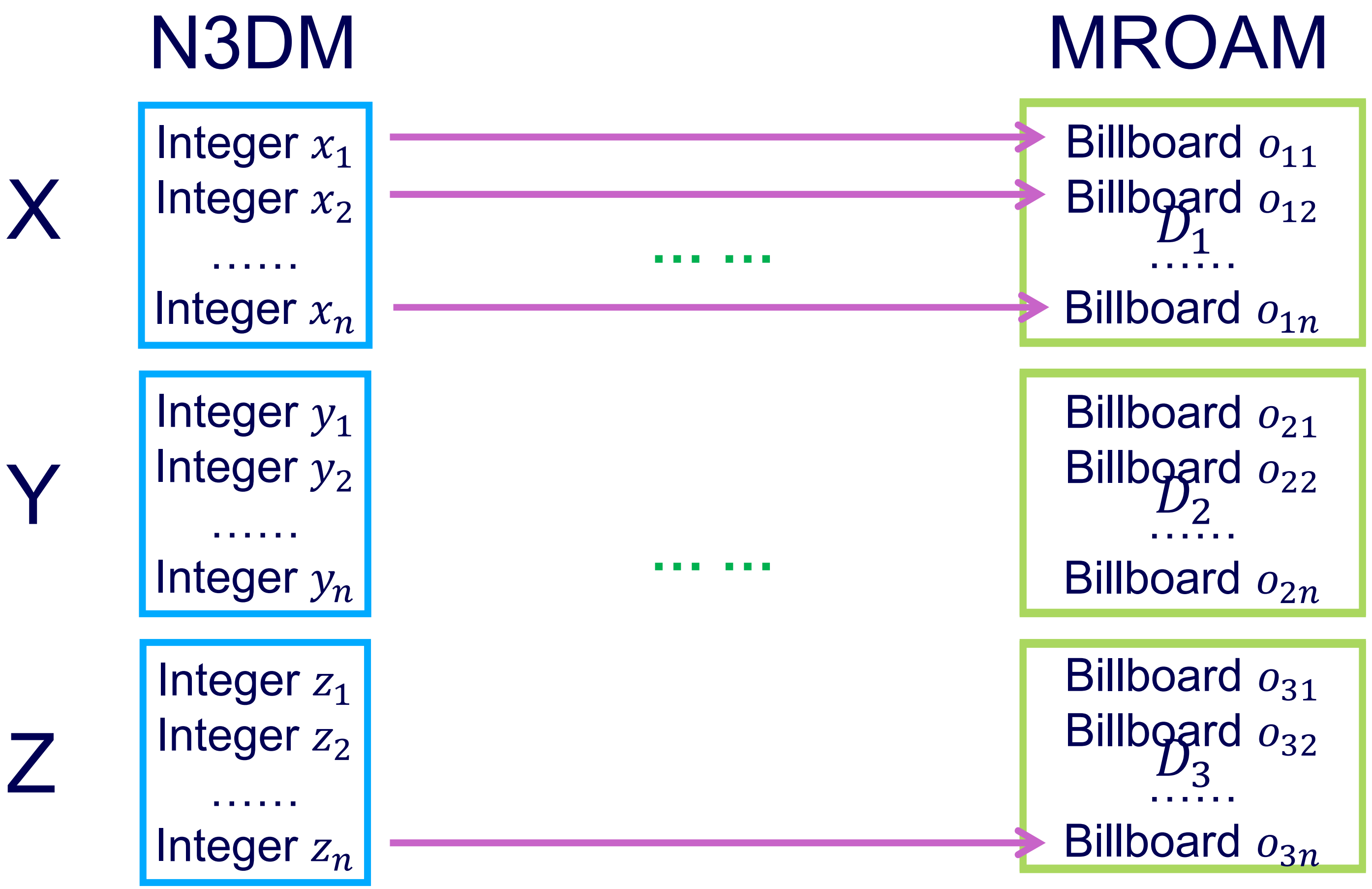
MROAM

D_1 | D_2 | D_3

Billboard Database

U, $|U| = 3n$

Hardness - Mapping



Hardness - Mapping

	N3DM		MROAM
X	Integer x_1 Integer x_2 …… Integer x_n	$I(o_{1i}) + I(o_{2j}) + I(o_{3k}) = I_1$ $I(o_{1i}) + I(o_{2j}) + I(o_{3k}) = I_2$	Billboard o_{11} Billboard o_{12} …… Billboard o_{1n}
Y	Integer y_1 Integer y_2 …… Integer y_n	n { ⋮	Billboard o_{21} Billboard o_{22} …… Billboard o_{2n}
Z	Integer z_1 Integer z_2 …… Integer z_n	$I(o_{1i}) + I(o_{2j}) + I(o_{3k}) = I_n$	Billboard o_{31} Billboard o_{32} …… Billboard o_{3n}

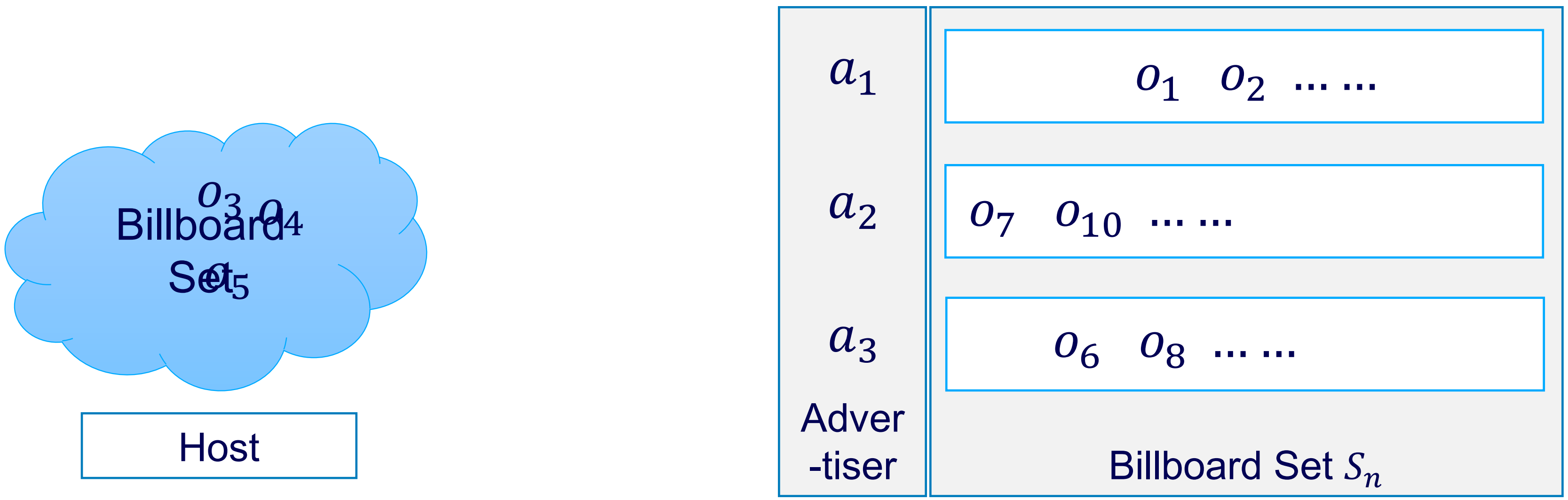
Hardness - Mapping

N3DM			MROAM	
X	Integer x_1 Integer x_2 …… Integer x_n	Find M Such that	Find plan S Such that	Billboard o_{11} Billboard o_{12} …… Billboard o_{1n}
Y	Integer y_1 Integer y_2 …… Integer y_n	For every triple $(x_i, y_j\ z_k) \in M$ $x_i + y_j + z_k = b$	For every triple $(o_{1i}, o_{2j}, o_{3k}) \in S$ $I(o_{1i}) + I(o_{2j})$ $+ I(o_{3k}) = I_n$	Billboard o_{21} Billboard o_{22} …… Billboard o_{2n}
Z	Integer z_1 Integer z_2 …… Integer z_n			Billboard o_{31} Billboard o_{32} …… Billboard o_{3n}

Synchronous Greedy

Iteratively assign a billboard o_i to S_n (belong to the advertiser a_n), such that o_i can maximize $\frac{R(S_n)-R(S_n \cup \{o_i\})}{I(\{o_i\})}$

Repeat until (1) Exhausting all billboards, or (2) all advertisers are satisfied



Synchronous Greedy

The problem of Synchronous Greedy:

The objective $R(S)$ of MROAM is neither monotone nor submodular.

Synchronous Greedy can easily produce a poor local minimum.

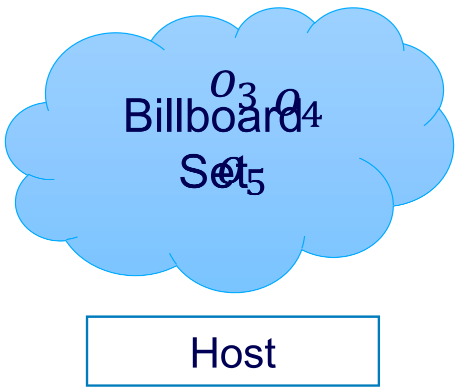
Randomized local search framework

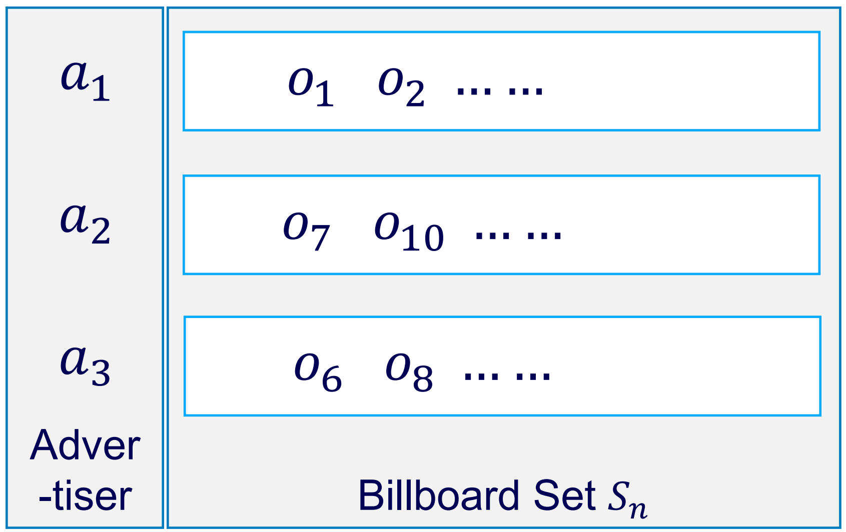
Advertiser-driven local search (ALS)

Billboard-driven local search (BLS)

Advertiser-driven local search (ALS)

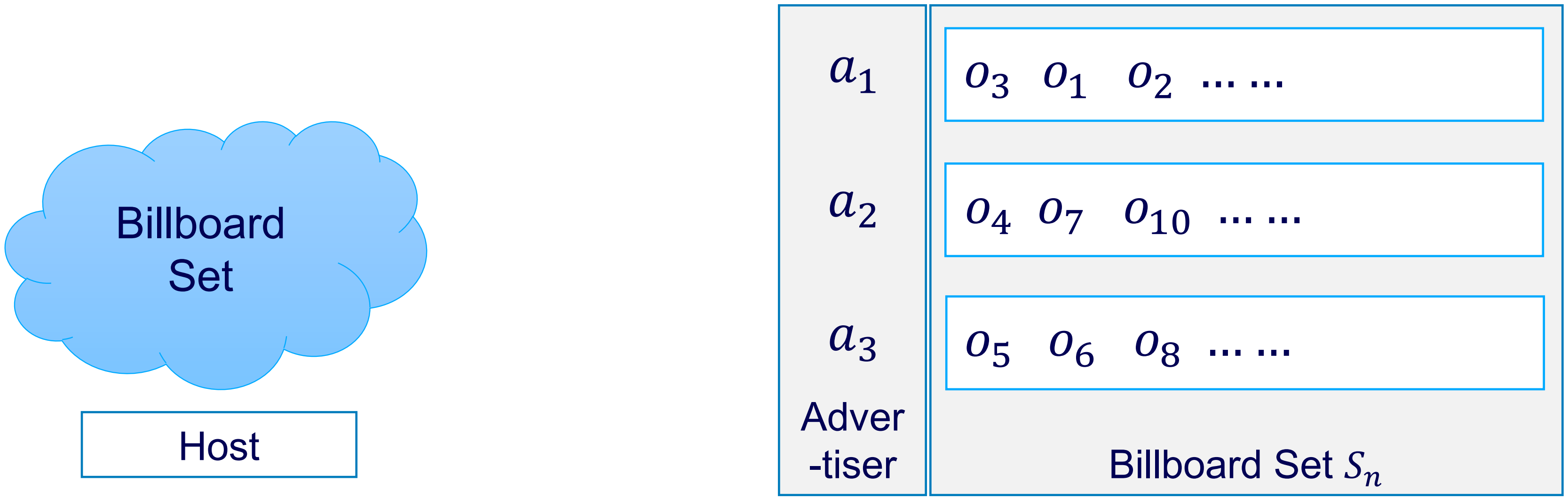
1. Greedy Randomized Adaptive phase (generating a solution).
   1. randomly assign a billboard to an advertiser
   2. execute a greedy search to assign the remaining billboards to the advertisers





Advertiser-driven local search (ALS)

1. Greedy Randomized Adaptive phase (generating a solution).
   1. randomly assign a billboard to an advertiser
   2. execute a greedy search to assign the remaining billboards to the advertisers
2. Advertiser-driven local search (finding a local minimum).



Advertiser-driven local search (ALS)

Billboards: $o_1 = \{t_1, \dots, t_{x-1}\}, o_2 = \{t_1, \dots, t_{x-2}, t_x\}, o_3 = \{t_x, t_{x+1}\}$

Advertiser	Budget L	Demand I_i	Billboard S_i	Influence $I(S_i)$	Regret
a_1	x	x	$\{o_1, o_2\}$	x	0
a_2	$x-1$	$x-1$	$\{o_3\}$	2	$x-1-2\gamma$

Exchange

Total Regret $x - 1 - 2\gamma$

$$R(S_i) = \begin{cases} L_i\left(1 - \dfrac{\gamma \cdot I(S_i)}{I_i}\right), & if \;\; a_i.L_i > I(S_i) \\ L_i\dfrac{I(S_i) - I_i}{I_i}, & otherwise \end{cases}$$

Advertiser-driven local search (ALS)

Billboards: $o_1 = \{t_1, \dots, t_{x-1}\}, o_2 = \{t_1, \dots, t_{x-2}, t_x\}, o_3 = \{t_x, t_{x+1}\}$

Advertiser	Budget L	Demand I_i	Billboard S_i	Influence $I(S_i)$	Regret
a_1	x	x	$\{o_3\}$	2	$x - 2\gamma$
a_2	$x - 1$	$x - 1$	$\{o_1, o_2\}$	x	1

Exchange

Total Regret $x + 1 - 2\gamma$

$$R(S_i) = \begin{cases} L_i\left(1 - \frac{\gamma \cdot I(S_i)}{I_i}\right), & if\ \ a_i.L_i > I(S_i) \\ L_i \frac{I(S_i) - I_i}{I_i}, & otherwise \end{cases}$$

Advertiser-driven local search (ALS)

Billboards: $o_1 = \{t_1, \dots, t_{x-1}\}, o_2 = \{t_1, \dots, t_{x-2}, t_x\}, o_3 = \{t_x, t_{x+1}\}$

Advertiser	Budget L	Demand I_i	Billboard S_i	Influence $I(S_i)$	Regret
a_1	x	x	$\{o_3, o_2\}$	x	0
a_2	$x-1$	$x-1$	$\{o_1\}$	$x-1$	0

$$R(S_i) = \begin{cases} L_i\left(1 - \dfrac{\gamma \cdot I(S_i)}{I_i}\right), & if\ \ a_i.L_i > I(S_i) \\ L_i\dfrac{I(S_i) - I_i}{I_i}, & otherwise \end{cases}$$

Total Regret 0 $x - 1 - 2\gamma$

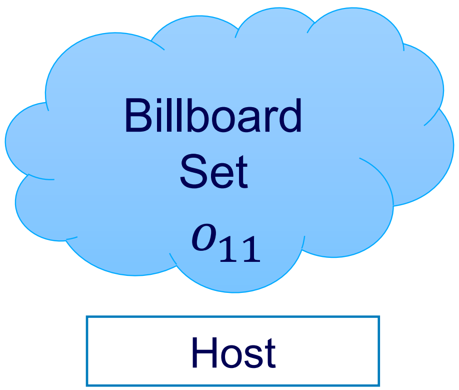
Randomized local search framework

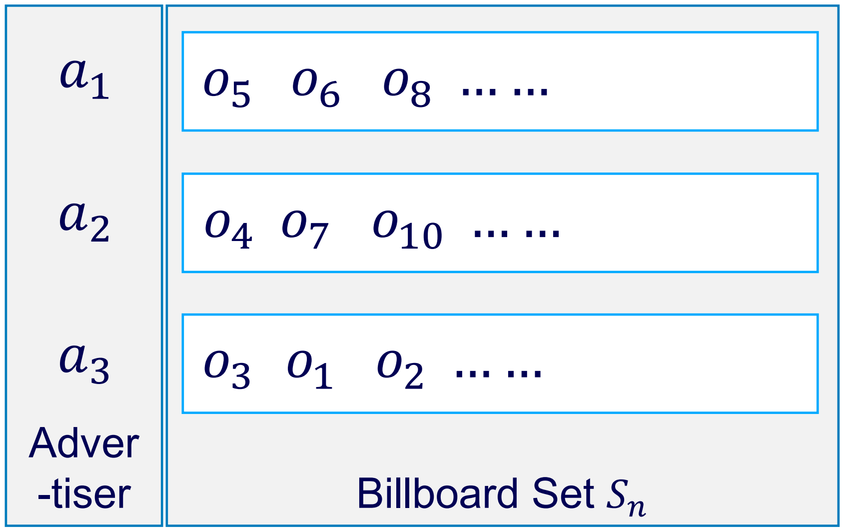
Advertiser-driven local search (ALS)

Billboard-driven local search (BLS)

Billboard-driven local search (BLS)

1. Greedy Randomized Adaptive phase (generating a solution).
   1. randomly assign a billboard to an advertiser
   2. execute a greedy search to assign the remaining billboards to the advertisers
2. Billboard-driven local search (BLS)
   1) Exchange within two advertisers
   2) Unassign one billboard
   3) Exchange with one unused billboard





Experiment

1. Evaluation algorithms' performance
2. What situation is good for the host?
    1. Demand-Supply Ratio α : Total demand / Host's supply
    2. Average-Individual Demand Ratio $p(\overline{I^A})$: Average demand / Host's supply

Individual \ Global	Low demand ($\alpha \leq 80\%$)	High demand ($\alpha \geq 100\%$)
Low demand ($p(\overline{I^{\mathcal{A}}}) \leq 2\%$)	Case 1	Case 3
High demand ($p(\overline{I^{\mathcal{A}}}) \geq 5\%$)	Case 2	Case 4

Experiment

City	Movement Pattern	Billboard Type
NYC	Taxi trajectories	Roadside billboards
SG	Public transport records	Bus station billboards

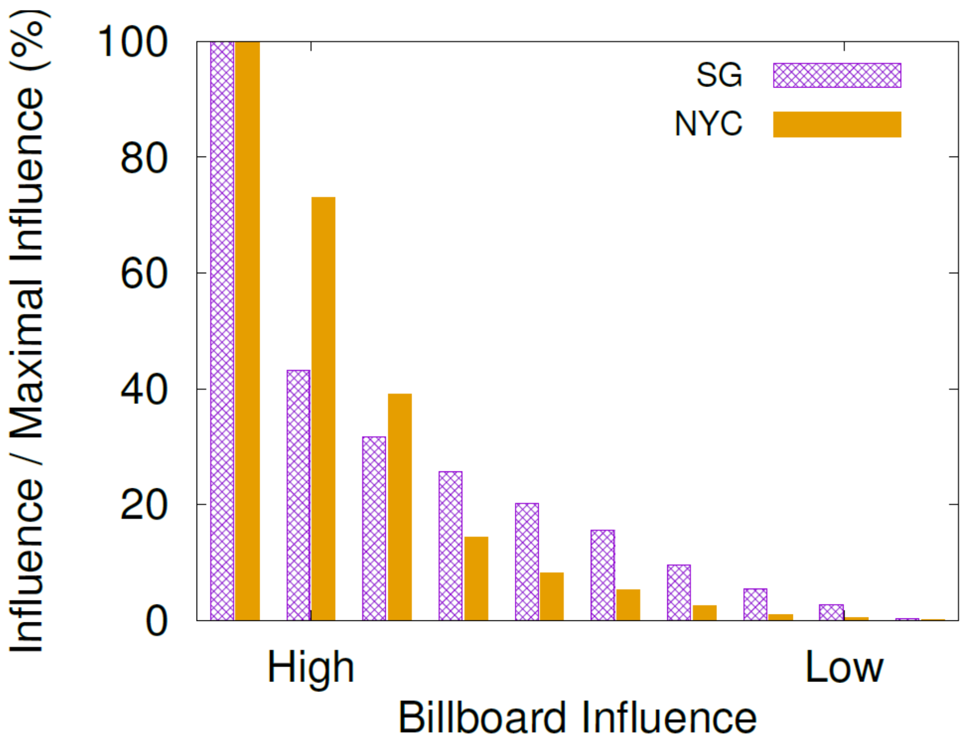
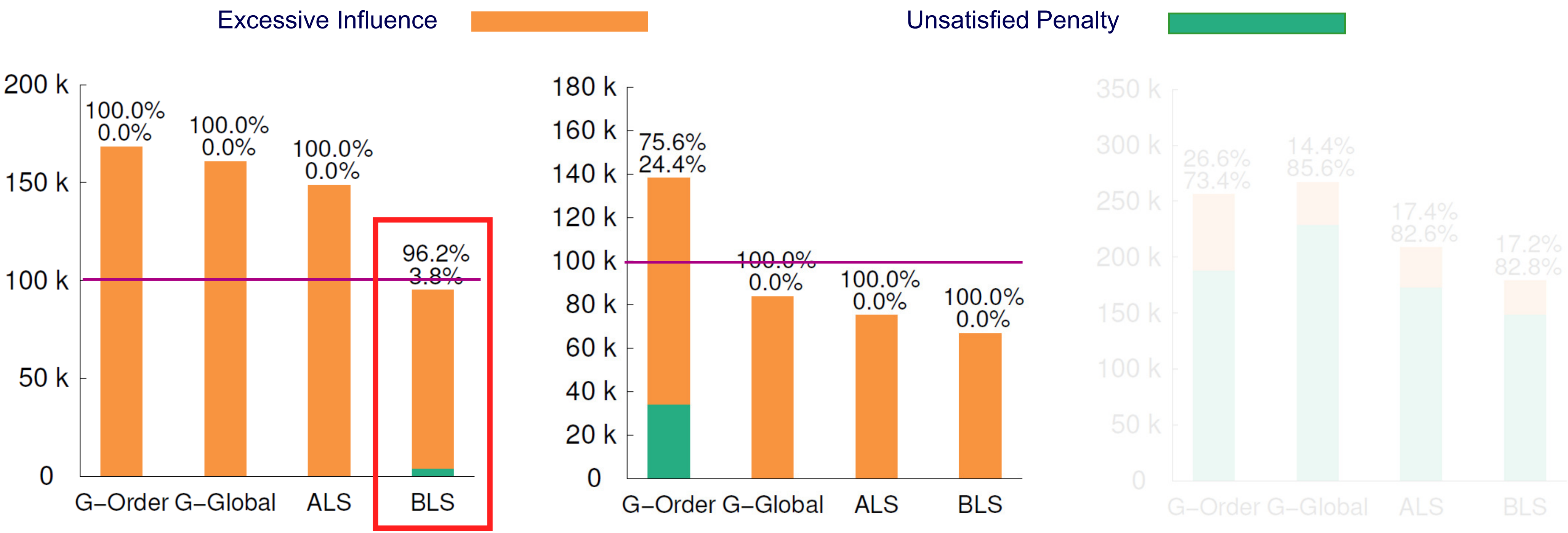


Figure 1: Influence Distribution of Billboards

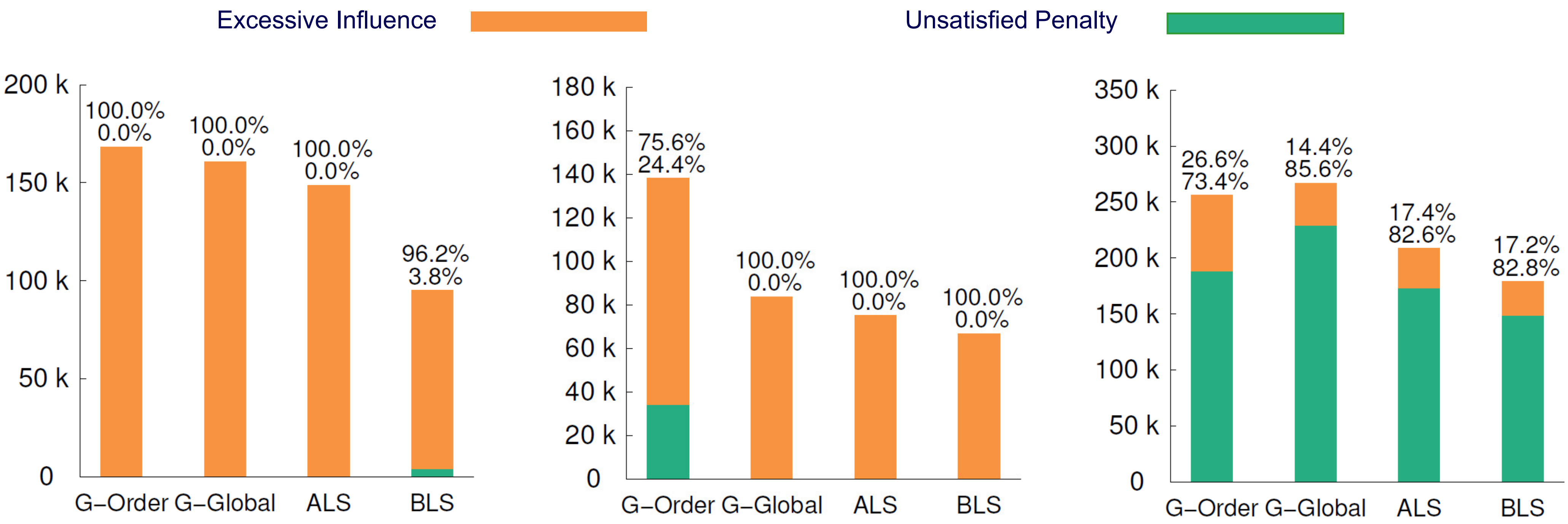
Low Individual Demand $p(\overline{I^A}) = 1\%$
- Low global demand vs. High global demand



(a) $\alpha = 40\%$ (b) $\alpha = 80\%$ (c) $\alpha = 120\%$

Figure 2: Regret of varying the demand-supply ratio α when $p(\overline{I^A}) = 1\%$ (NYC)

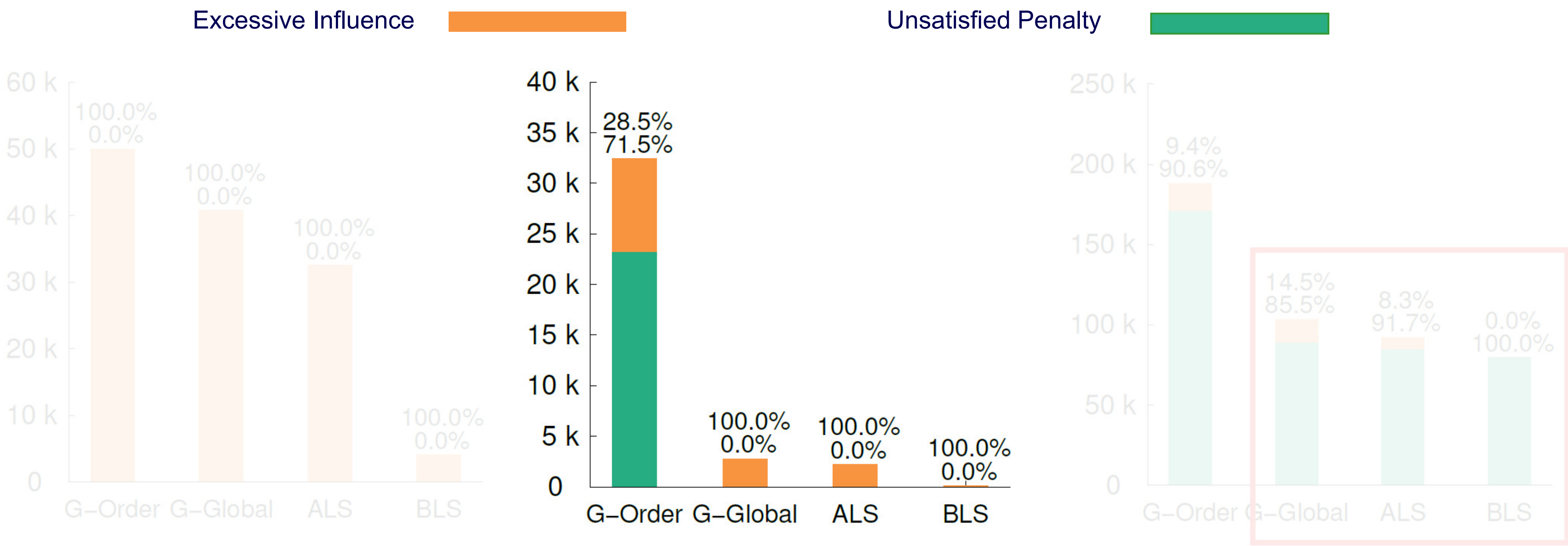
Low Individual Demand $p(\overline{I^A}) = 1\%$
- Low global demand vs. High global demand



(a) $\alpha = 40\%$ (b) $\alpha = 80\%$ (c) $\alpha = 120\%$

Figure 2: Regret of varying the demand-supply ratio α when $p(\overline{I^A}) = 1\%$ (NYC)

High Individual Demand $p(\overline{I^A}) = 10\%$
- Low global demand vs. High global demand

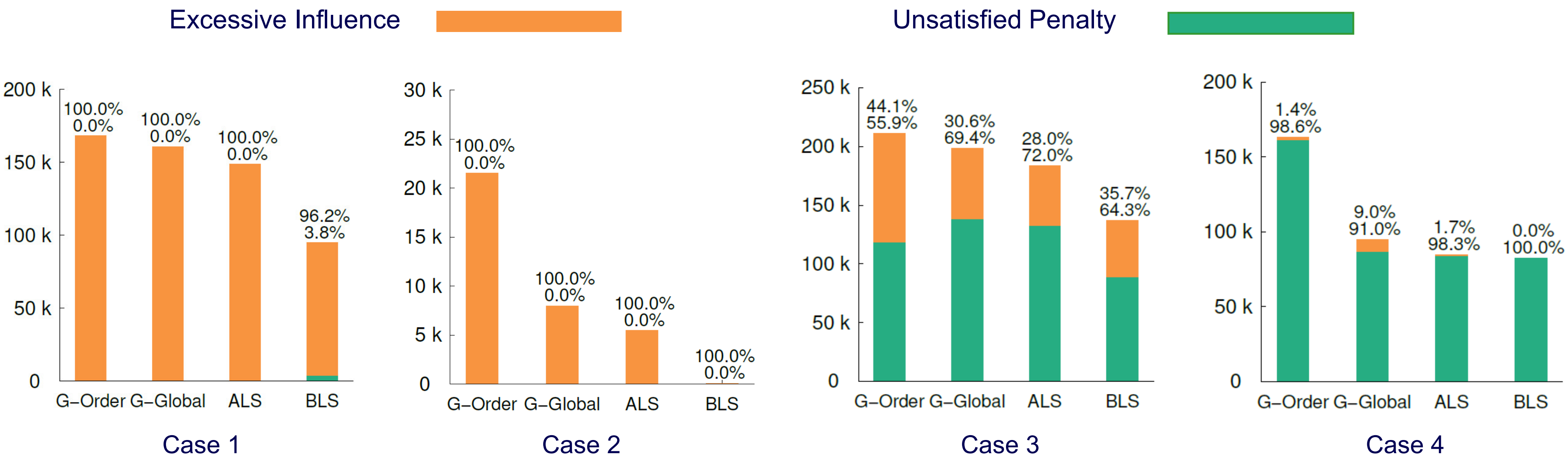


(a) $\alpha = 40\%$ (b) $\alpha = 80\%$ (c) $\alpha = 120\%$

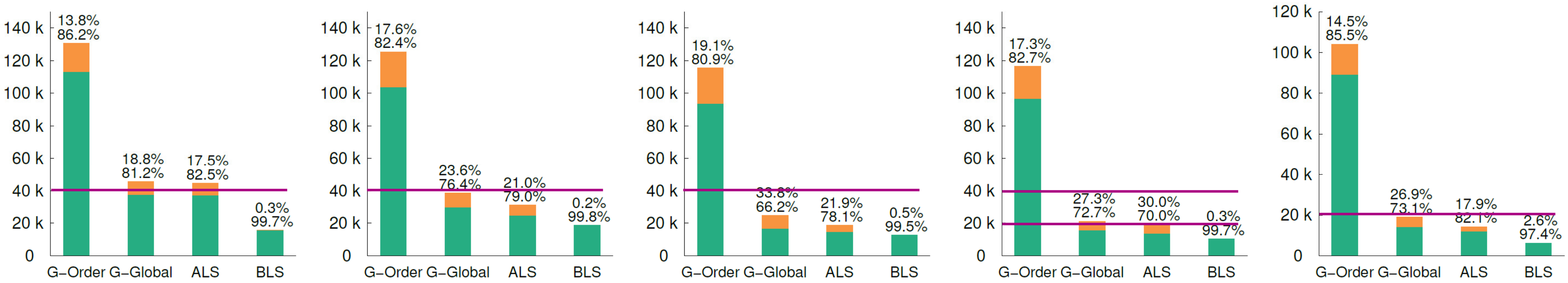
Figure 3: Regret of varying the demand-supply ratio α when $p(\overline{I^A}) = 10\%$ (NYC)

Experiment

Global / Individual	Low demand ($\alpha \leq 80\%$)	High demand ($\alpha \geq 100\%$)
Low demand ($p(\overline{I^{\mathcal{A}}}) \leq 2\%$)	Case 1	Case 3
High demand ($p(\overline{I^{\mathcal{A}}}) \geq 5\%$)	Case 2	Case 4



Unsatisfied Penalty Ratio γ



(a) $\gamma = 0$ (b) $\gamma = 0.25$ (c) $\gamma = 0.50$ (d) $\gamma = 0.75$ (e) $\gamma = 1$

Figure 4: Regret of varying the unsatisfied penalty ratio γ (NYC)

$$R(S_i) = \begin{cases} L_i\left(1 - \dfrac{\gamma \cdot I(S_i)}{I_i}\right), & if\ \ a_i.L_i > I(S_i) \\ L_i \dfrac{I(S_i) - I_i}{I_i}, & otherwise \end{cases}$$

Unsatisfied Penalty

Excessive Influence

NYC - SG

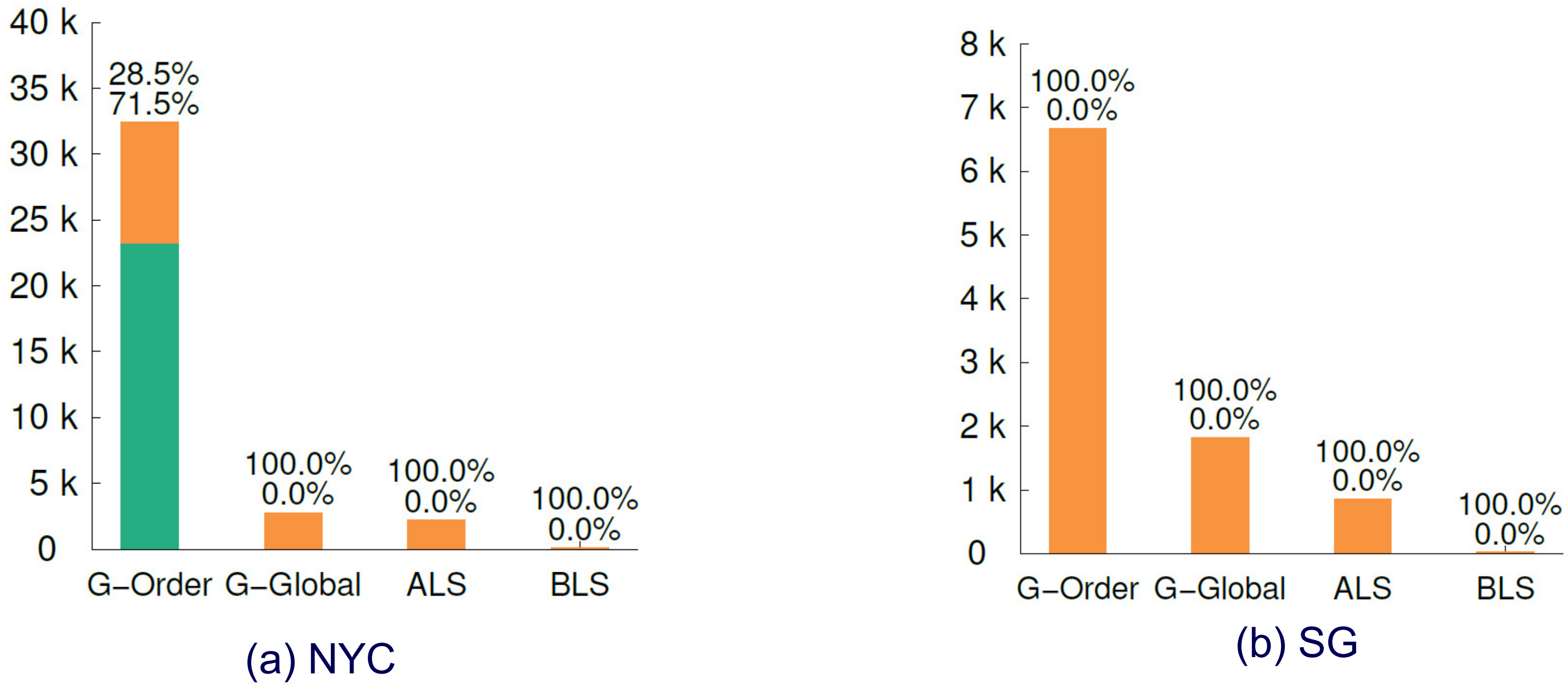


Figure 5: Regret when $\alpha = 80\%$ and $p(\overline{I^A}) = 10\%$

Thanks

NYC - SG

The influence range is modeled as a circle centered on a billboard with a radius of λ meters

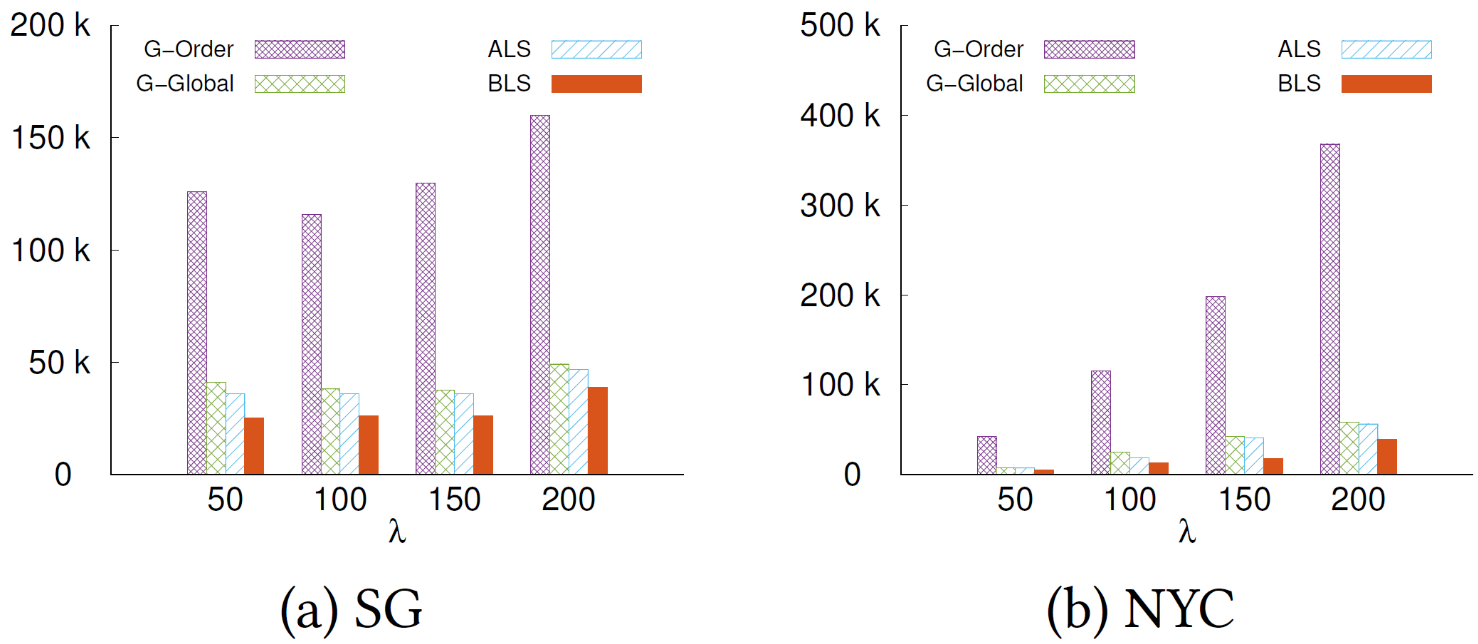


Figure 6: Regret of varying λ

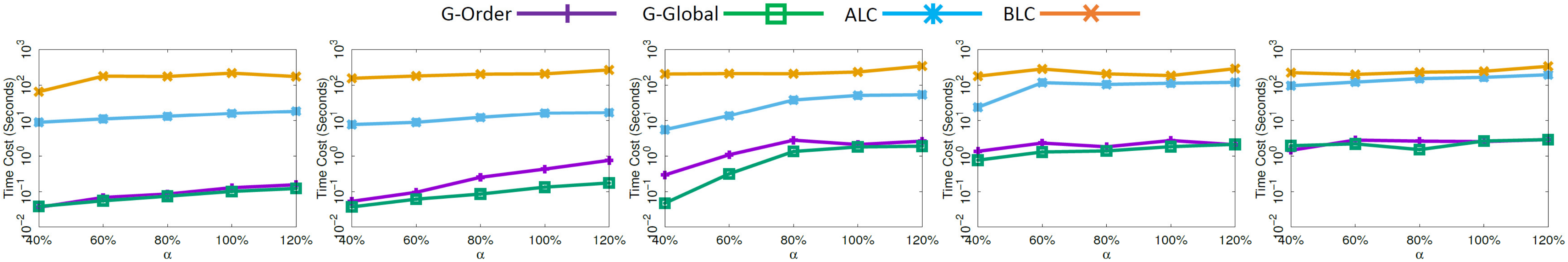
Dataset

Table 5: Statistics of Datasets

	$\lvert\mathcal{T}\rvert$	$\lvert\mathcal{U}\rvert$	AvgDistance	AvgTravelTime
NYC	1.7×10^6	1462	2.9km	569s
SG	2.2×10^6	4092	4.2km	1342s

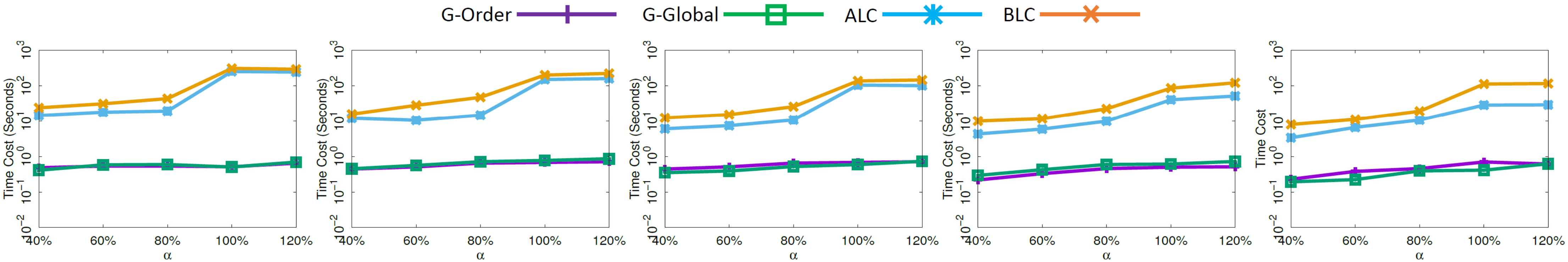
Table 6: Parameter Settings

Parameter	Values
α	40%, 60%, 80%, **100%**, 120%
$p(\overline{I^{\mathcal{A}}})$	1%, 2%, **5%**,10%, 20%
γ	0, 0.25, **0.5**, 0.75, 1
λ	50m, **100m**, 150m, 200m



(a) $p(\overline{I^{\mathcal{A}}}) = 1\%$ ($|\mathcal{A}| = 100$) (b) $p(\overline{I^{\mathcal{A}}}) = 2\%$ ($|\mathcal{A}| = 50$) (c) $p(\overline{I^{\mathcal{A}}}) = 5\%$ ($|\mathcal{A}| = 20$) (d) $p(\overline{I^{\mathcal{A}}}) = 10\%$ ($|\mathcal{A}| = 10$) (e) $p(\overline{I^{\mathcal{A}}}) = 20\%$ ($|\mathcal{A}| = 5$)

Figure 8: Efficiency Study (SG)



(a) $p(\overline{I^{\mathcal{A}}}) = 1\%$ ($|\mathcal{A}| = 100$) (b) $p(\overline{I^{\mathcal{A}}}) = 2\%$ ($|\mathcal{A}| = 50$) (c) $p(\overline{I^{\mathcal{A}}}) = 5\%$ ($|\mathcal{A}| = 20$) (d) $p(\overline{I^{\mathcal{A}}}) = 10\%$ ($|\mathcal{A}| = 10$) (e) $p(\overline{I^{\mathcal{A}}}) = 20\%$ ($|\mathcal{A}| = 5$)

Figure 9: Efficiency Study (NYC)

Algorithm 1: Budget-Effective Greedy

Input: $\mathcal{U}$, $\mathcal{T}$, $\mathcal{A}$

Output: S

Order each advertiser $a_i \in \mathcal{A}$ based on descending order of $L_i / \mathcal{I}_i$

Initialize $S \leftarrow \{S_1, ..., S_{|\mathcal{A}|}\}$

foreach $a_i \in \mathcal{A}$ **do**

  while $\mathcal{U} \neq \emptyset \wedge \mathcal{I}_i > I(S_i)$ **do**

    Select $o \in \mathcal{U}$ that maximizes $\frac{R(S_i) - R(S_i \cup \{o\})}{I(\{o\})}$

    $S_i \leftarrow S_i \cup \{o\}$

    $\mathcal{U} \leftarrow \mathcal{U} \setminus \{o\}$

return S

Algorithm 2: Synchronous Greedy

Input: $\mathcal{U}, \mathcal{T}, \mathcal{A}, S^{in}$ $(S^{in} = \{S_1^{in}, \ldots, S_{|\mathcal{A}|}^{in}\})$
Output: S

$S \leftarrow S^{in}$
while *TRUE* **do**
  foreach $a_i \in \mathcal{A}$ **do**
    if $\mathcal{I}_i > I(S_i)$ **then**
      if $\mathcal{U} \neq \emptyset$ **then**
        Select $o \in \mathcal{U}$ that maximizes $\frac{R(S_i) - R(S_i \cup \{o\})}{I(\{o\})}$
        $S_i \leftarrow S_i \cup \{o\}$
        $\mathcal{U} \leftarrow \mathcal{U} \setminus \{o\}$
  if *more than two* $a_i \in \mathcal{A}$ *are not satisfied* **then**
    Release $S_j \in S$ such that $\mathcal{I}_j > I(S_j)$ and has minimum $L_j / \mathcal{I}_j$
    $\mathcal{A} \leftarrow \mathcal{A} \setminus \{a_j\}$
  else
    return S

Algorithm 3: Randomized Local Search

Input: $\mathcal{U}, \mathcal{T}, \mathcal{A}$

Output: S^{best}

$S^{best} \leftarrow$ SynchronousGreedy($\mathcal{U}, \mathcal{T}, \mathcal{A}, \emptyset$)

while *the number of iterations < a preset count* **do**

  $\mathcal{U}^* \leftarrow \mathcal{U}$

  for $a_i \in \mathcal{A}$ **do**

    $S_i \leftarrow$ {a random billboard $o \in \mathcal{U}^*$}

    $\mathcal{U}^* \leftarrow \mathcal{U}^* \setminus \{o\}$

  $S \leftarrow \{S_1, ..., S_{|\mathcal{A}|}\}$

  $S^* \leftarrow$ SynchronousGreedy($\mathcal{U}^*, \mathcal{T}, \mathcal{A}, S$)

  $S^{can} \leftarrow$ Advertiser-drivenLocalSearch($\mathcal{U}^*, \mathcal{T}, S^*$)

  if $R(S^{can}) < R(S^{best})$ **then**

    $S^{best} \leftarrow S^{can}$

return S^{best}

Algorithm 4: Advertiser-driven Local Search (ALS)

Input: $\mathcal{U}$, $\mathcal{T}$, S^{best}

Output: S^{best}

while *TRUE* **do**
    $S^{can} \leftarrow S^{best}$
    foreach $a_i \in \mathcal{A}$ **do**
        foreach $a_j \in \mathcal{A} \setminus \{a_i\}$ **do**
            if *Exchange* S_i *with* S_j *will reduce* $R(S^{can})$ **then**
                Exchange S_i with S_j
    if $R(S^{can}) < R(S^{best})$ **then**
        $S^{best} \leftarrow S^{can}$
    else
        return S^{best}

Algorithm 5: Billboard-driven Local Search (BLS)

Input: $\mathcal{U}$, $\mathcal{T}$, S^{best}

Output: S^{best}

while *TRUE* **do**
    $S^{can} \leftarrow S^{best}$
    foreach $S_i \in S^{can}$ **do**
        foreach $S_j \in S^{can} \setminus S_i$ **do**
            if $\exists o_m \in S_i \wedge o_n \in S_j$ *such that Exchange*(o_m, o_n) *will decrease* $R(S^{can})$ **then**
                Exchange(o_m, o_n)
        if $\exists o_m \in S_i \wedge o_n \in \mathcal{U}$ *such that Exchange*(o_m, o_n) *will decrease* $R(S^{can})$ **then**
            Exchange(o_m, o_n)
        if $\exists o_m \in S_i$ *such that releasing* o_m *will decrease* $R(S^{can})$ **then**
            Release $o_m \in S_i$
    $S \leftarrow$ SynchronousGreedy$(\mathcal{U}, \mathcal{T}, \mathcal{A}, S^{can})$
    if $R(S) < R(S^{can})$ **then**
        $S^{can} \leftarrow S$
    The same as Lines 4.7-4.10

Hardness - N3DM to MROAM

Numerical 3-Dimensional Matching (N3DM)

Input:

1. A bound b (set demanded influence I_i as b)
2. Three multisets of integers X, Y and Z, $|X| = |Y| = |Z| = n$
   (set advertiser database A, such that $|A| = n$)

Find:

Matching relation M, such that

1. Every integer in X, Y and Z occurs exactly once
2. Every triple $(x_i, y_j, z_k) \in M$, $x_i + y_j + z_k = b$ hold

Seek Influence – Our Work

Minimizing Regret for the OOH Advertising Market problem (MROAM)

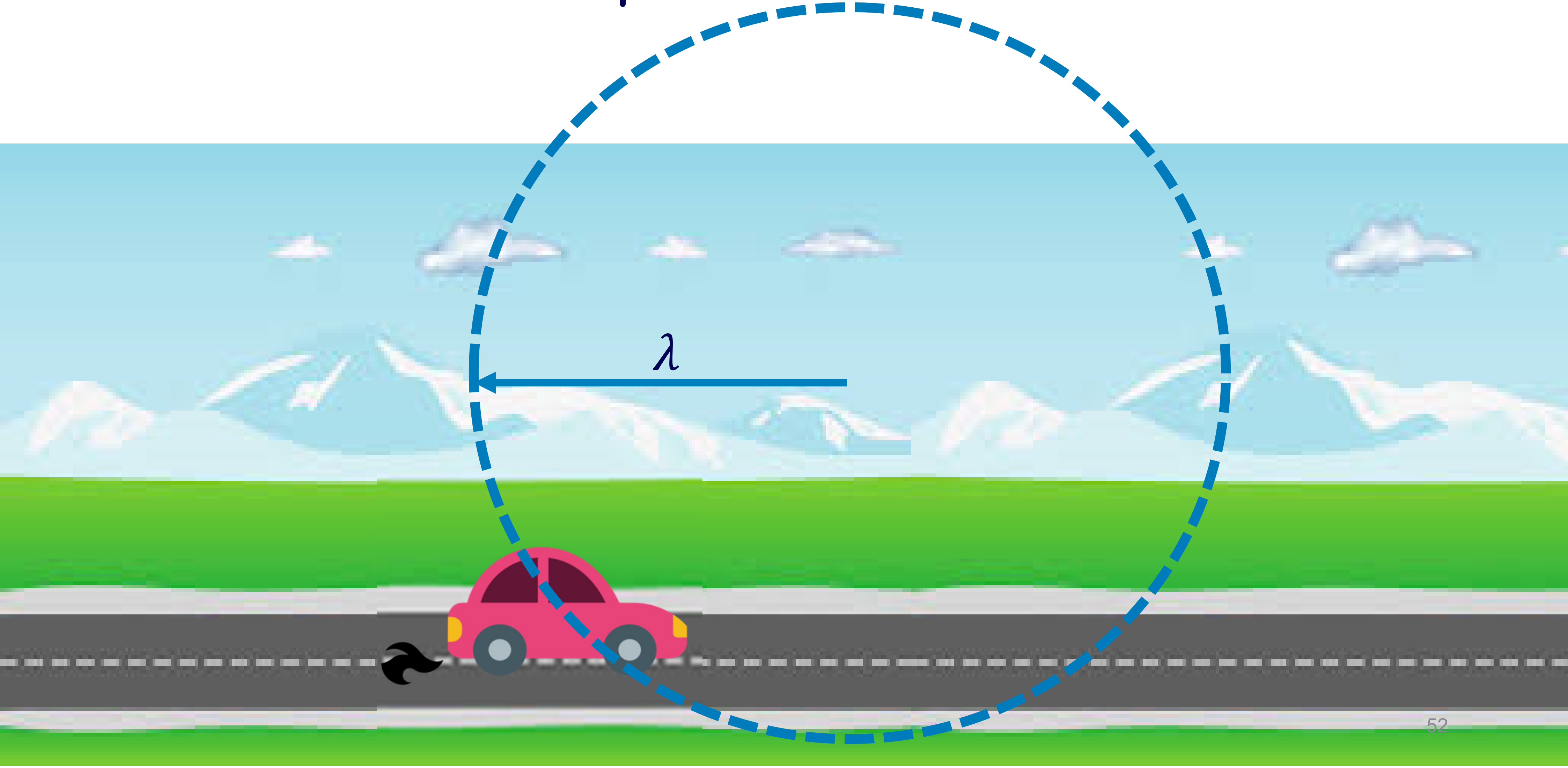
Input

1. Billboard database U
2. Trajectory database T
3. Advertiser set $A = \{a_1, \dots, a_{|A|}\}$ with demanded influence I_i and a payment L_i
4. Influence Measurement $I(S_i)$
5. Regret Measurement $R(S_i)$

Output

Billboard deployment strategy $S = \{S_1, \dots, S_{|A|}\}$ that minimizes the regret of the influence host

1. How a billboard impresses an audience?

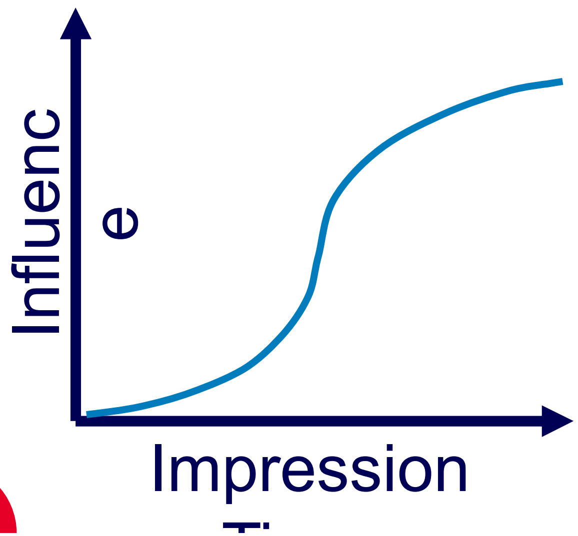


Different Influence Measurement

1. One impression model

$$p(S,t) = 1 - \prod_{o_i \in S} [1 - pr(o_i, t)]$$

2. Multiple impression model



$$p(S,t) = \begin{cases} \frac{1}{1+exp\{\alpha - \beta \cdot \Sigma_{o_i \in S} I(o_i,t)\}} & if \ \ \exists o_i \in S \ I(o_i,t) = 1 \\ 0 & otherwise \end{cases}$$

$$I(S) = \sum_{t_j \in T} p(S, t_j)$$

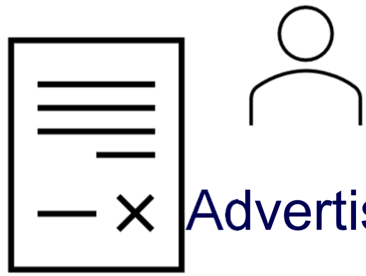
For Advertiser – Existing Work

I have Billboards
I provide Billboards
I earn Money

Host

Billboard	Influence	Cost
Billboard 1	30	100
Billboard 2	50	200
Billboard 3	100	300
……		

I pay Money
I rent Billboards
(To maximize influence)



Advertiser

Contract
Request: I want to rent Billboard 3.
Payment: I will pay $300.